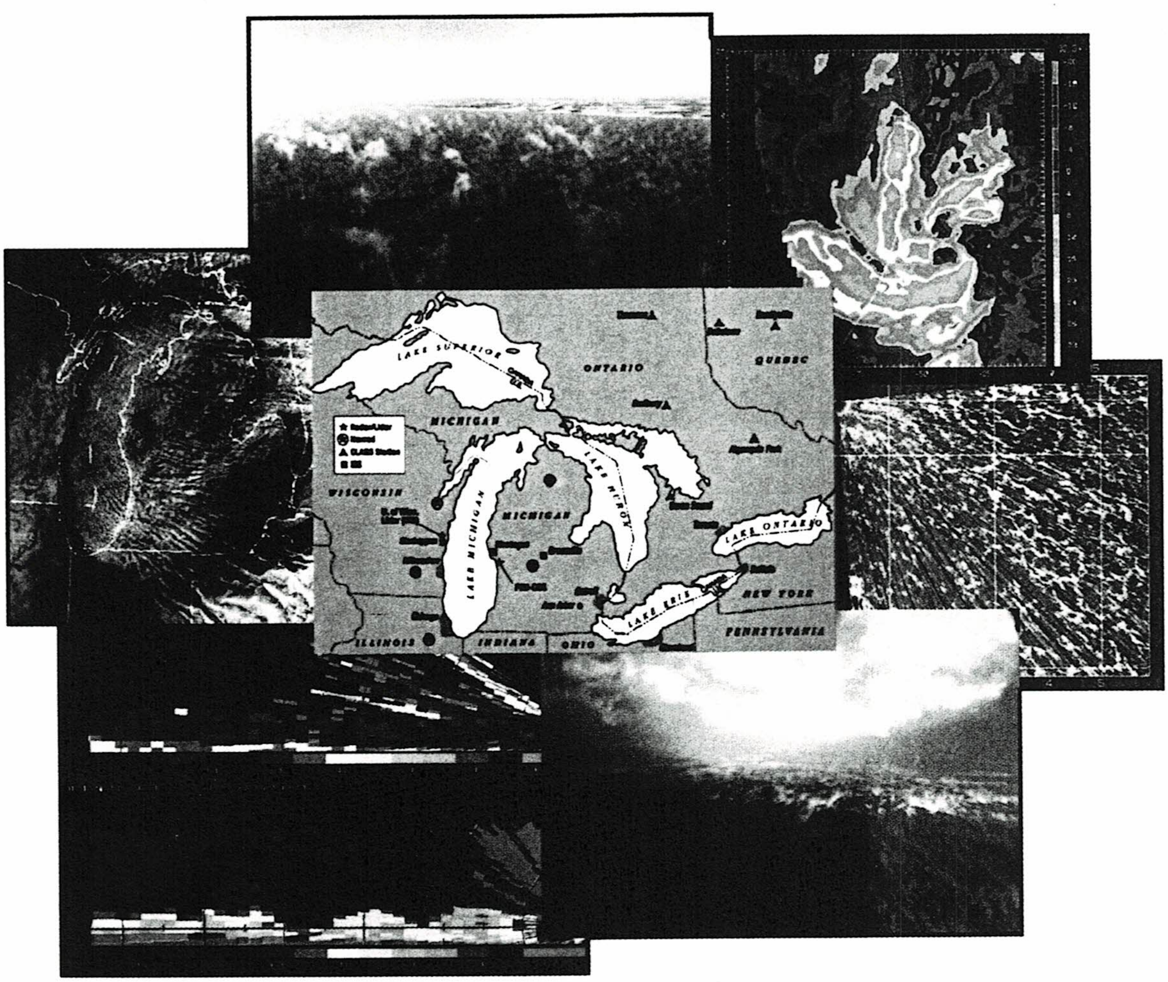
14th Symposium on Boundary Layer and Turbulence

7–11 August 2000 Aspen, CO



AMERICAN METEOROLOGICAL SOCIETY

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LOW-ORDER MODELS OF A SHEARED CONVECTIVE BOUNDARY LAYER

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1. INTRODUCTION

Mesoscale shallow convection (MSC) results from a complex mix of various processes: convection, vertical shear of horizontal wind, dynamical, thermal, and entrainment instabilities, effects of rotation, etc. As discussed in a review by Atkinson and Zhang (1996), the roles of these processes in the evolution of MSC remain unclear, thus "a gap exists in our understanding of the dynamics of the PBL".

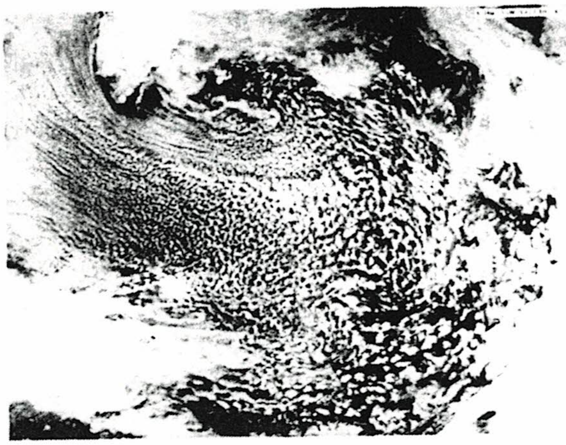

Figure 1. Cold air moving southeastward from the frozen Baffin Bay region to the south of Greenland and over the open waters of the North Atlantic (Scorer 1986).

Generally considered as the atmospheric manifestation of Rayleigh–Bénard convection, MSC occurs in two distinctive regimes: 2D rolls, or cloud streets, and 3D cells; the latter regime is called mesoscale cellular convection (MCC) (Agee 1987; Atkinson and Zhang 1996). Both are characterized by convective depths of 1 to 3 km and wide ranges of aspect ratios (2 to 20 for rolls and 5 to 50 for cells).

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It has generally been thought that the conditions for formation of rolls in the atmosphere are a moderate surface heat flux and a strong wind speed. Observational data during cold air outbreaks over the west Pacific area showed that under almost the same temperature and humidity profiles, rolls occurred when the vertical wind shear was 1 to 10 m s^{-1} km^{-1}, while cells existed when the shear was less than these values (Miura 1986). In their large-eddy simulation, Sykes and Henn (1989) also observed the tendency of the imposed shear to reorganize the convective cells into 2D rolls. Fig. 1 demonstrates a typical occurrence of rolls that give way to cells down the flow. However, recent lidar images obtained by Eloranta et al. (1999) indicate that there might be a misunderstanding of the role of wind shear. On the western shoreline of Lake Michigan, they observed 3D open cells just offshore where the wind shear should be greatest and 2D structures out over the lake where it should be less.

In this study, low-order models (LOMs) will be formulated that may help to clarify the roles and interplay of convection and shear in the dynamics of MSC. The LOMs are constructed following a general approach to the development of physically sound LOMs in the form of coupled Volterra gyrostats (Gluhovsky and Agee 1997, Gluhovsky and Tong 1999).

The Volterra (1899) gyrostat (also Wittenburg 1977)

$$\begin{aligned} \dot{v}_1 &= p v_2 v_3 + b v_3 - c v_2 , \\ \dot{v}_2 &= q v_3 v_1 + c v_1 - a v_3 , \\ \dot{v}_3 &= r v_1 v_2 + a v_2 - b v_1 ; \end{aligned} \tag{1}$$

where p, q, r, a, b, c are constants, $p+q+r=0$, describes precisely certain fluid dynamical situations (Gluhovsky, 1982; Gluhovsky and Tong, 1999). The simplest gyrostat in a forced regime,

$$\begin{aligned} \dot{v}_1 &= -q v_2 v_3 \qquad -\gamma_1 v_1 + f, \\ \dot{v}_2 &= \ \ q v_3 v_1 - a v_3 \ -\gamma_2 v_2, \\ \dot{v}_3 &= \qquad\qquad a v_2 \ -\gamma_3 v_3, \end{aligned} \tag{2}$$

becomes, after a linear change of variables (Gluhovsky 1982), the Lorenz (1963) model for 2D Rayleigh–Bénard convection.

$(\alpha_4 = \alpha_2,\ \alpha_5 = \alpha_3)$, that consists of two coupled gyrostats (4) describing dynamics in the (x, z) and (y, z) planes, respectively. For coefficients in this system and systems (8) and (9) below, we found their analytical expressions in terms of the parameters of Eqs. (3).

2.2 *The Simplest Model of 3D Convection with Shear*

To allow for the generation of spontaneous shear, two symmetry breaking modes for each component of horizontal velocity are added. This extends model (7) to a system of four coupled gyrostats

$$
\begin{aligned}
\dot{x}_1 &= -x_2x_3 - x_4x_5 - \alpha_1 x_1 + f,\\
\dot{x}_2 &= x_3x_1 - x_3 - \alpha_2 x_2,\\
\dot{x}_3 &= x_2 + px_6x_7 - \alpha_3 x_3,\\
\dot{x}_4 &= x_5x_1 - x_5 - \alpha_4 x_4,\\
\dot{x}_5 &= x_4 + px_8x_9 - \alpha_5 x_5,\\
\dot{x}_6 &= qx_7x_3 - \alpha_6 x_6,\\
\dot{x}_7 &= rx_3x_6 - \alpha_7 x_7,\\
\dot{x}_8 &= qx_9x_5 - \alpha_8 x_8,\\
\dot{x}_9 &= rx_5x_8 - \alpha_9 x_9.
\end{aligned}
\qquad (8)
$$

In Eqs. (8), $\alpha_4 = \alpha_2,\ \alpha_5 = \alpha_3,\ \alpha_6 = \alpha_8,\ \alpha_7 = \alpha_9$. The two additional gyrostats in system (8) are, in fact, Euler gyroscopes with coefficients p, q, r computed from parameters of Eqs. (3). Note that system (8) provides the gyrostatic form for a model of convection in an electrically conducting fluid (Kennett 1976) and models of convection of shear flows in tokamak plasmas (Bazdenkov and Pogutse 1993, Aoyagi et al. 1997, Takayama et al. 1998). Aoyagi et al. (1997) also discuss their model in relation to Rayleigh–Bénard convection with shear. All models considered in these papers can be converted to the form of Eqs. (8), just with different coefficients in each case.

2.3 *A 3D Analog of the improved Howard-Krishnamurti model of convection with shear*

Adding 3 more temperature modes produces the 3D analog of the Howard – Krishnamurti (1986) model improved by Thiffeault and Horton (1996), which was discussed in the Introduction. In this new model, two more "Lorenz" gyrostats (those with coefficients *d*) are added to system (8), as well as two degenerate gyroscopes (those with coefficients *c*):

$$
\begin{aligned}
\dot{x}_1 &= -x_2x_3 - x_4x_5 - \alpha_1 x_1 + f_1,\\
\dot{x}_2 &= x_3x_1 - x_3 - cx_6x_{10} - \alpha_2 x_2,\\
\dot{x}_3 &= x_2 + px_6x_7 - \alpha_3 x_3,\\
\dot{x}_4 &= x_5x_1 - x_5 - cx_8x_{11} - \alpha_4 x_4,\\
\dot{x}_5 &= x_4 + px_8x_9 - \alpha_5 x_5,\\
\dot{x}_6 &= qx_7x_3 - \alpha_6 x_6,\\
\dot{x}_7 &= rx_3x_6 + dx_{10} - \alpha_7 x_7,\\
\dot{x}_8 &= qx_9x_5 - \alpha_8 x_8,\\
\dot{x}_9 &= rx_5x_8 + dx_{11} - \alpha_9 x_9,\\
\dot{x}_{10} &= dx_{12}x_7 - dx_7 + cx_2x_6 - \alpha_{10} x_{10},\\
\dot{x}_{11} &= dx_{12}x_9 - dx_9 + cx_4x_8 - \alpha_{11} x_1,\\
\dot{x}_{12} &= -dx_7x_{10} - dx_9x_{11} - \alpha_{12} x_{12} + f_2.
\end{aligned}
\qquad (9)
$$

In Eqs. (9), $\alpha_4 = \alpha_2,\ \alpha_5 = \alpha_3,\ \alpha_6 = \alpha_8,\ \alpha_7 = \alpha_9,\ \alpha_{10} = \alpha_{11}$.

2.4 *Conserved Quantities*

For all LOMs developed above, expressions for kinetic energy *K*, potential energy *U*, and available potential energy *A*, were obtained. In the absence of forcing and friction, the total energy $K + U$, and the "unavailable potential energy", $U - A$, are expected to be constants. Then $K + A$ is also conserved. It can be easily checked that the sum of squares of all variables, which is a linear combination of $K + U$ and $K + A$ (thus, representing some form of energy), is conserved for all systems of coupled gyrostats.

It was also demonstrated that the most successful LOMs in problems of geophysical fluid dynamics are, in fact, coupled Volterra gyrostats (Gluhovsky 1986, Gluhovsky and Agee 1997). In these models, linear gyrostatic terms (linear terms in Eqs. (1)) occur due to various factors peculiar to geophysical fluid dynamics, such as stratification, rotation, and topography. When such models are expanded by increasing the order of approximation or by adding new physical mechanisms, they still have the structure of coupled gyrostats.

Coupled gyrostats possess the fundamental conservation properties of the Navier-Stokes equations, while failure for a LOM to have a gyrostatic structure usually indicates that proper energy integrals are not conserved (Gluhovsky and Tong, 1999). One example is provided by the important Howard – Krishnamurti (1986) model that has received particular attention in studies of Rayleigh–Bénard convection with shear. Howard and Krishnamurti noticed that the model possessed trajectories going to infinity that they rightly attributed to deficiencies of the truncation. Indeed, Thiffeault and Horton (1996) found that the model lacks energy conservation in the dissipationless limit, which can be remedied by adding one term in the Galerkin expansion of the temperature. This operation was demonstrated to add terms to the original model permitting to transform it into a system of coupled gyrostats thereby ensuring energy conservation and boundedness of trajectories (Gluhovsky and Tong 1999).

Thus, giving LOMs a gyrostatic structure ensures that certain significant physics from the original equations of atmospheric dynamics is retained.

2. LOW-ORDER MODELS FOR 3D RAYLEIGH-BÉNARD CONVECTION WITH SHEAR

The third dimension is crucial for understanding MSC. Therefore, instead of the stream function formulation (in Boussinesq approximation) generally employed in the 2D case, the following nondimensional system based on the equation for the vorticity $\varsigma = \nabla \times \mathbf{v}$ was used:

$$\frac{\partial \varsigma}{\partial t} = (\varsigma \cdot \nabla)\mathbf{v} - (\mathbf{v} \cdot \nabla)\varsigma + \nu \nabla^2 \varsigma + \left(\hat{\mathbf{x}} \frac{\partial \theta}{\partial y} - \hat{\mathbf{y}} \frac{\partial \theta}{\partial x} \right),$$

$$\frac{\partial \theta}{\partial t} = -\mathbf{v} \cdot \nabla \theta + v_z + \kappa \nabla^2 \theta, \quad \nabla \cdot \mathbf{v} = 0, \tag{3}$$

All variables in Eqs. (3) are dimensionless forms of $\mathbf{v}$ (velocity), ς (vorticity), θ (temperature deviation from stable profile), ν (kinematic viscosity), and κ (thermal conductivity). Also, t is time and x, y, z are the spatial coordinates. Stress-free boundary conditions are adopted: at the top and bottom of the layer $(z = 0, \pi)$,

$$v_z = 0, \quad \frac{\partial v_x}{\partial z} = 0, \quad \frac{\partial v_y}{\partial z} = 0, \quad \theta = 0,$$

and at $x = y = 0,\ \pi / a$ (a is the aspect ratio),

$$\frac{\partial \theta}{\partial x} = \frac{\partial \theta}{\partial y} = v_x = v_y = 0.$$

The celebrated Lorenz (1963) model results from Eqs. (3) with 2D constraints, by employing the maximally truncated Galerkin expansions

$$\begin{aligned} \theta &= X_1(t)\sin(2z) + X_2(t)\cos(ax)\sin(z), \\ v_x &= X_3(t)\sin(ax)\cos(z), \\ v_z &= -aX_3(t)\cos(ax)\sin(z), \end{aligned} \tag{4}$$

Its gyrostatic equivalent (2) can be further simplified by a linear change of variables

$$x_1 = (a/q)v_1 + 1 - (a/q)^2, \quad x_2 = (a/q)v_2, \quad x_3 = v_3, \quad t' = qt,$$

to take the form

$$\begin{aligned} \dot{x}_1 &= \Big| -x_2x_3 & &\Big| -\alpha_1 x_1 + f, \\ \dot{x}_2 &= \Big| \quad x_3x_1 - x_3 & &\Big| -\alpha_2 x_2, \\ \dot{x}_3 &= \Big| \quad x_2 & &\Big| -\alpha_3 x_3, \end{aligned} \tag{5}$$

where $\alpha_i = \gamma_i / q$, $f = aF/q^2 + (1-(a/q)^2)(\gamma_1/q)$. Again, the variables $X_i(t)$ in Eqs. (4) and $x_i(t)$ in Eqs. (5) are linearly related (Gluhovsky 1982).

2.1 *A 3D Analog of the Lorenz Model*

The Galerkin expansions for the 3D case corresponding to (4),

$$\begin{aligned} \theta &= X_1(t)\sin(2z) + X_2(t)\cos(ax)\sin(z) \\ &\qquad + X_4(t)\cos(ay)\sin(z), \\ v_x &= X_3(t)\sin(ax)\cos(z), \\ v_y &= X_5(t)\sin(ay)\cos(z), \\ v_z &= -aX_3(t)\cos(ax)\sin(z) - aX_5(t)\cos(ay)\sin(z), \end{aligned} \tag{6}$$

produce a 3D analog of the Lorenz (1963) model

$$\begin{aligned} \dot{x}_1 &= \Big| -x_2x_3 \quad \Big| -x_4x_5 \quad \Big| -\alpha_1 x_1 + f \\ \dot{x}_2 &= \Big| \quad x_3x_1 - x_3 \quad \Big| \quad \Big| -\alpha_2 x_2 \\ \dot{x}_3 &= \Big| \quad x_2 \quad \Big| \quad \Big| -\alpha_3 x_3 \\ \dot{x}_4 &= \quad \Big| \quad x_5x_1 - x_5 \quad \Big| -\alpha_4 x_4 \\ \dot{x}_5 &= \quad \Big| \quad x_4 \quad \Big| -\alpha_5 x_5 \end{aligned} \tag{7}$$

3. SUMMARY AND CONCLUSIONS

In this paper, *3D* low-order models for convection with shear were developed in the form of systems of coupled gyrostats. The advantage of coupled gyrostats is that such systems possess conservation properties of the original equations (in the dissipationless limit), thus permitting sound physical behavior.

As noted by Brown and Chua (1992), "there is a pressing need for new nonlinear techniques that employ a building block approach whereby simple well-understood components are used to construct models of complex dynamical systems". We believe that coupled gyrostats could play the role of the above building blocks in problems of geophysical fluid dynamics and turbulence.

The study of the models' behavior is in progress, and results may be available at the time of the conference.

4. ACKNOWLEDGMENTS

The authors are grateful to E. M. Agee for useful discussions. This work was supported by NSF grant ATM-9909009 and by Purdue Research Foundation Grant 690/1396-2979.

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